Mark Scheme (Results)

Summer 2017

Pearson Edexcel GCE Mathematics

Core Mathematics 2 (6664/01)

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Publications Code xxxxxxxx*

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper or ag- answer given
- ⎿ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>1.</td><td>$\left(3-\frac{1}{3}x\right)^5$ -
$3^5 + {}^5C_1 3^4(-\frac{1}{3}x) + {}^5C_2 3^3(-\frac{1}{3}x)^2 + {}^5C_3 3^2(-\frac{1}{3}x)^3 ...$
First term of 243
$\left({}^5C_1 \times ... \times x\right) + \left({}^5C_2 \times ... \times x^2\right) + \left({}^5C_3 \times ... \times x^3\right)...$
$=(243 ..)\ -\frac{405}{3}x + \frac{270}{9}x^2 - \frac{90}{27}x^3...$
$=(243 ..) -135x + 30x^2 - \frac{10}{3}x^3..$</td><td>

B1
M1
A1
A1
(4)
[4]</td></tr>
<tr><td>Alternative method</td><td>$\left(3-\frac{1}{3}x\right)^5 = 3^5(1-\frac{x}{9})^5$
$3^5(1 + {}^5C_1(-\frac{1}{9}x) + {}^5C_2(-\frac{1}{9}x)^2 + {}^5C_3(-\frac{1}{9}x)^3\ ...)$
Scheme is applied exactly as before</td><td></td></tr>
<tr><td></td><td colspan="2">

Notes

B1: The constant term should be 243 in their expansion
M1: Two of the three binomial coefficients must be correct and must be with the correct power of x. Accept ${}^5C_1$ or $\binom{5}{1}$ or 5 **as a coefficient, and** ${}^5C_2$ or $\binom{5}{2}$ or 10 **as another and** ${}^5C_3$ or $\binom{5}{3}$ or 10 **as another……..** Pascal's triangle may be used to establish coefficients. NB: If they only include the first two of these terms then the M1 may be awarded.
A1: Two of the final three terms correct – may be unsimplified i.e. two of $-135x + 30x^2 - \frac{10}{3}x^3$ correct, or two of $-\frac{405}{3}x + \frac{270}{9}x^2 - \frac{90}{27}x^3$ (may be just two terms)
A1: All three final terms correct and simplified. (Can be listed with commas or appear on separate lines. Accept in reverse order.) Accept correct alternatives to $-\frac{10}{3}$ e.g. $-3\frac{1}{3}$ or $-3.\dot{3}$ the recurring must be clear. 3.3 is not acceptable. Allow e.g. $+-135x$</td></tr>
<tr><td></td><td colspan="2">e.g. The common error $3^5 + {}^5C_1 3^4(-\frac{1}{3})x + {}^5C_2 3^3(-\frac{1}{3})x^2 + {}^5C_3 3^2(-\frac{1}{3})x^3 = (243) -135x - 90x^2 - 30x^3$ would earn B1, M1, A0, A0, so 2/4
If extra terms are given then isw
No negative signs in answer also earns B1, M1, A0, A0
If the series is divided through by 3 at the final stage after an error or omission resulting in all multiple of three coefficients then apply scheme to series before this division and ignore subsequent work (isw)
Special Case: Only gives first three terms $=(243 ..) -135x + 30x^2$ ... or $243 - \frac{405}{3}x + \frac{270}{9}x^2$
Follow the scheme to give B1 M1 A1 A0 special case. (Do not treat as misread.)
Answers such as $243\ + 405 - \frac{1}{3}x + 270 - \frac{1}{9}x^2 + 90 - \frac{1}{27}x^3$.. gain no credit as the binomial coefficients are not linked to the x terms.</td></tr>
</table>

Question Number	Scheme	Marks
2.	$\frac{\sin x}{16} = \frac{\sin 50^0}{13}$	M1
	$(\sin x) = \frac{16 \times \sin 50}{13}$ $(= 0.943 \text{ but accept } 0.94)$	A1
	x = awrt 70.5(3) **and** 109.5 **or** 70.6 and 109.4	dM1 A1 **(4)** **[4]**
	Notes M1: use sine formula correctly **in any form.** Allow awrt 0.77 for $\sin 50°$ A1: give the correct value or correct expression for sin x (this implies the M1 mark). If it is given as expression they do not need degrees symbol. $\frac{\sin 50 \times 16}{13}$ is fine, If this is given as a decimal allow answers which round to 0.94. Allow awrt −0.323 (radians) here but no further marks are available. If they give this as x (not sin$x$) and do not recover this is A0 dM1: Correct work leading to x=… (via inverse sin) expression or value for sin$x$ If the previous A mark has been awarded for a correct expression then this is for getting to awrt 70.5 or 109.5 (allow for 70.6 or 109.4). If the previous A mark was not gained, e.g. rounding errors were made in rearranging the correct sine formula then award dM1 for evidence of use of **inverse sin in degrees** on their value for sin$x$ (may need to check on calculator). **NB** 70.5 following a correct sine formula will gain M1A1M1. A1: deduce and state **both of the answers** x = 70.5 and 109.5 (do not need degrees) Accept awrt these. Also accept 70.6 and 109.4. (Second answer is sometimes obtained by a long indirect route but still scores A1) If working in radians throughout, answers are 1.23 and 1.91 and this can be awarded M1 A1 M1 A0 (Working with 50 radians gives probable answers of -0.3288 and 3.47 giving M1A1M0A0) **Special case:** Wrong labelling of triangle. This simplifies the problem as there is only one solution for angle x. So it is not treated as a misread. If they find the missing side as awrt 12.6 then proceed to find an angle or its sine or cosine then give M1A0M0A0	
	Alternative Method using cosine rule Let $BC = a$. M1: uses the cosine rule to form to form a three term quadratic equation in a (e.g. $a^2 - 32a\cos 50^\circ + 87 = 0$ or $a^2 - \text{awrt}20.6a + 87 = 0$ though allow slips in signs rearranging) A1: Solves and obtains a correct value for a of awrt 14.6 **or** awrt 5.95. dM1: A **correct full** method to find (at least) one of the two angles. May use cosine rule again, or find angle BAC and then use sine rule. As in the main scheme, if the previous A mark has been awarded then they should obtain one of the correct angles for this mark. A1: deduces both correct answer as in main scheme. **NB** obtaining only one correct angle will usually score M1A1M1A0 in any method.	

Question Number	Scheme	Marks
3.	<table><tr><td>x</td><td>0</td><td>0.5</td><td>1</td><td>1.5</td><td>2</td></tr><tr><td>y</td><td>1</td><td>2.821</td><td>**6**</td><td>12.502</td><td>26.585</td></tr></table>	
(a)	$\{\text{At } x=1,\}\ y=6$ (allow 6.000 or even 6.00)	B1 cao
		(1)
(b)	$\frac{1}{2}\times 0.5$;	B1 oe
	$\{1+26.585+2(2.821+\text{their } 6+12.502)\}$ For structure of $\{\ldots\ldots\ldots\ldots\ldots\}$;	M1A1ft
	$\frac{1}{2}\times 0.5\,\{1+26.585+2(2.821+6+12.502)\}\ \{=\frac{1}{4}(70.231)=17.557..\}$ = awrt 17.56	A1
		(4)
(c)	10 + "17.56" = "27.56"	B1ft
		(1)
		[6]

Notes

(a) B1: 6

(b) B1: for using $\frac{1}{2}\times 0.5$ or $\frac{1}{4}$ or equivalent.

M1: requires the correct $\{\ldots\ldots\}$ bracket structure. It needs the first bracket to contain first y value **plus** last y value and the second bracket to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). M0 if values used in brackets are x values instead of y values

A1ft: for the correct bracket $\{\ldots\ldots\}$ following through candidate's y value found in part (a).

A1: for answer which rounds to 17.56

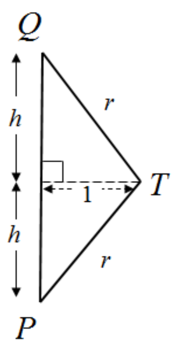
NB: Separate trapezia may be used: B1 for 0.25, M1 for 1/2 $h(a+b)$ used 3 or 4 times (and A1ft if it is all correct) Then A1 as before.

Special case: Bracketing mistake $0.25\times(1+26.585)+2(2.821+\text{their } 6+12.502)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). An answer of 49.542 usually indicates this error.

(c) B1ft: 10 + their answer to part (b)
(May be obtained by using the trapezium rule again with all values for y increased by 5)

Question Number	Scheme	Marks
4. (a)	**Usually answered in radians: Uses** $BCD = 3.5\times(angle), = 3.5\times1.77 = 6.195$ (m) (accept awrt 6.20)	M1 A1 **(2)**
(b)	Area = $\frac{1}{2}(3.5)^2\times1.77 = 10.84$ (m^2)	M1 A1 **(2)**
(c)	Area of triangle = $\frac{1}{2}\times3.7\times3.5\times\sin(\text{angle})$, = $\frac{1}{2}\times3.7\times3.5\times\sin(\frac{\pi}{2}-\frac{1.77}{2})$ (=awrt 4.1)	M1, A1
	Total area = $"10.84"+2\times"4.101"$	M1
	= 19.04	A1cao **(4)** **[8]**
	Notes	
(a)	M1: uses $s = 3.5\times\theta$ with θ in radians or completely correct work in degrees	
	A1: awrt 6.20 or just 6.2 (do not need to see units) Correct answer can imply the method.	
(b)	M1 for attempt to use $A = \frac{1}{2}\times3.5^2\times\theta$ (Accept θ in degrees.)	
	A1 for awrt 10.84 (do not need to see units) isw if correct answer is followed by 10.8. Correct answer can imply the method.	
(c)	M1: Uses area of triangle $\frac{1}{2}\times3.7\times3.5\times\sin(\text{angle})$ Must be correct method for area of triangle but may be less direct.	
	A1: Correct expression using correct angle i.e. $\frac{\pi}{2}-\frac{1.77}{2}$ or awrt 0.69 or awrt 39 degrees (need at least 2 sf if no other work seen, but may be implied by correct final answer) If correct expression is given then isw (so e.g. isw an answer of 0.0775 implying angle set to degrees on calculator)	
	M1: Adds **twice** their **second calculated area** (even if rectangle or segment) to their sector area (may have been slips or errors in one or both formulae – such as missing ½ or mixture of degrees and radians or weak attempt at triangle area) so M0A0M1A0 is a possible mark distribution	
	A1: 19.04 cao (common answer through insufficient accuracy is 19.08 which loses final mark)	
	Special Case. The mark profile M1A0M1A0M1A0M1A0 can be given if the angle is misunderstood as 1.77π or as *AFB* for example	
	If $"10.84"+3.5\times3.7\sin(\text{angle})$ is used then this can gain both M marks and the A marks if correct.	
	But use of $3.5\times3.7\sin(\text{angle})$ and later doubled and added to "10.84" is 1st M0, 2nd M1.	

Question number	Scheme	Marks
5	$x^2 + y^2 - 10x + 6y + 30 = 0$	
(a)	Uses any appropriate method to find the coordinates of the centre, e.g achieves $(x\pm5)^2 + (y\pm3)^2 = \ldots$. Accept (±5,±3) as indication of this.	M1

	Centre is $(5, -3)$.	A1	
			(2)
(b) Way 1	Uses $(x \pm "5")^2 - "5^2" + (y \pm "3")^2 - "3^2" + 30 = 0$ to give		
	$r = \sqrt{"25"+"9"-30}$ or $r^2 = "25"+"9"-30$ (not $30-25-9$)	M1	
	$r = 2$	A1cao	
			(2)
Or Way 2	Using $\sqrt{g^2+f^2-c}$ from $x^2+y^2+2gx+2fy+c=0$ (Needs formula stated or correct working)	M1	
	$r = 2$	A1	
			(2)
(c) Way 1	Use $x = 4$ in *an* equation of circle and obtain equation in y only	M1	
	e.g $(4-5)^2+(y+3)^2=4$ or $4^2+y^2-10\times4+6y+30=0$		
	Solve their quadratic in y and obtain **two** solutions for y	dM1	
	e.g. $(y+3)^2=3$ or $y^2+6y+6=0$ so $y=-3\pm\sqrt{3}$	A1	
			(3)
Or Way 2	 Divide triangle *PTQ* and use Pythagoras with $"r"^2-("5"-4)^2=h^2$,	M1	
	Find h and evaluate $"-3"\pm h$. May recognise $(1, \sqrt{3}, 2)$ triangle.	dM1	
	So $y = -3\pm\sqrt{3}$	A1	
			(3)
			[7]

	Notes
(a)	**Parts (a) and (b) can be marked together** **M1** as in scheme and can be implied by $(\pm5,\pm3)$ May be awarded for writing LHS as $(x\pm5)^2 + (y\pm3)^2 = ...$ or by comparing with $x^2+y^2+2gx+2fy+c=0$ to write down centre $(-g,-f)$ directly **A1:** (5, −3). **This correct answer implies M1A1**
(b)	**M1** for a **full** correct method leading to $r = \ldots$, or $r^2 =$ with their 5, their −3, their 25 and their 9 and their "−30". Completion of square method errors result in **M0** here. Usually r = 4 or r = 16 imply M0A0 **A1** 2 cao Do not accept $r=\pm2$ unless it is followed by (r =) 2 The correct answer with no wrong work seen implies M1A1 **Special case**: if centre is given as (-5, -3) or (5, 3) or (-5, 3) allow **M1A1** for r = 2 worked correctly. i.e. $r^2 = "25"+"9"-30$
(c)	**M1**: *Way 1*: Use x = 4 in a circle equation (may have wrong centre and/or radius) to obtain an equation in y only or *Way* 2. Uses geometry to find equation in h (ft on their radius and centre) **dM1**: (needs first method mark) Solve their quadratic in y or *Way* 2. Uses their h and their y coordinate correctly **A1**: cao

Question Number	Scheme	Marks
6. (a)	Attempt $f(3)$ or $f(-3)$ Use of long division is M0A0 as factor theorem was required. $f(-3) = 162 - 63 - 120 + 21 = 0$ so $(x + 3)$ is a factor	M1 A1 **(2)**
(b)	Either (Way 1): $f(x) = (x+3)(-6x^2 + 11x + 7)$ $= (x+3)(-3x+7)(2x+1)$ or $-(x+3)(3x-7)(2x+1)$	M1A1 M1A1 **(4)**
	Or (Way 2) Uses trial or factor theorem to obtain $x = -1/2$ **or** $x = 7/3$ Uses trial or factor theorem to obtain both $x = -1/2$ **and** $x = 7/3$ Puts three factors together (see notes below) Correct factorisation : $(x+3)(7-3x)(2x+1)$ or $-(x+3)(3x-7)(2x+1)$ oe	M1 A1 M1 A1 **(4)**
	Or (Way 3) No working three factors $(x+3)(-3x+7)(2x+1)$ otherwise need working	M1A1M1A1 (4)
(c)	$2^y = \frac{7}{3}, \rightarrow \log(2^y) = \log\left(\frac{7}{3}\right)$ or $y = \log_2\left(\frac{7}{3}\right)$ or $\frac{\log(7/3)}{\log 2}$ $\{y = 1.222392421...\} \Rightarrow y =$ awrt 1.22	B1, M1 A1 **(3)** **[9]**
	Notes	
(a)	M1 for attempting either $f(3)$ or $f(-3)$ – with **numbers substituted into expression** A1 for calculating $f(-3)$ **correctly to 0**, and they must state $(x + 3)$ **is a factor** for A1 (or equivalent ie. QED, □ or a tick). A conclusion may be implied by a preamble, "if f(−3) = 0, (x+3) is a factor". $-6(-3)^3-7(-3)^2 + 40(-3) + 21 = 0$ so $(x + 3)$ is a factor of $f(x)$ is M1A1 providing bracketing is correct.	
(b)	1st M1: attempting to divide by $(x+3)$ leading to a 3TQ beginning with the correct term, usually $-6x^2$. This may be done by a variety of methods including long division, comparison of coefficients, inspection etc. Allow for work in part (a) if the result is used in (b). 1st A1: usually for $(-6x^2 + 11x + 7)$ … Credit when seen and use isw if miscopied 2nd M1: for a ***valid**** attempt to factorise their quadratic (* see notes on page 6 - General Principles for Core Mathematics Marking section 1) 2nd A1 is cao and needs all three factors together fully factorised. Accept e.g. $-3(x+3)(x-\frac{7}{3})(2x+1)$ but $(x+3)(x-\frac{7}{3})(-6x-3)$ and $(x+3)(3x-7)(-2x-1)$ are A0 as not fully factorised. Ignore subsequent work (such as a solution to a quadratic equation.) Way 2: The second M mark needs three roots together so $\pm 6(x-\alpha)(x-\beta)(x+3)$ or equivalent where they obtained α and β by trial, so if correct roots identified, then $(x+3)(3x-7)(2x+1)$ can gain M1A1M1A0. N.B. Replacing $(-6x^2 + 11x + 7)$ (already awarded M1A1) by $(6x^2 - 11x - 7)$ giving $(x+3)(3x-7)(2x+1)$ can have M1A0 for factorization so M1A1M1A0	
(c)	B1: $2^y = \frac{7}{3}$ M1: Attempt to take logs to solve $2^y = \alpha$ or $2^y = 1/\alpha$, where $\alpha > 0$ and α was a root of their factorization. A1: for an answer that rounds to 1.22. If other answers are included (and not "rejected") such as ln(−3) or −1 lose final A mark **Special case:** Those who deal **throughout** with $f(x) = 6x^3 + 7x^2 - 40x - 21$ They may have full credit in part (a). In part (b) they can achieve a maximum of M1A0M1A0 unless they return the negative sign to give the correct answer. This is then full marks. Part (c) is fine. So they could lose 2 marks on the factorisation. (Like a misread)	

Question Number	Scheme	Marks
7. (i)	Use of power rule so $\log(x+a)^2 = \log 16a^6$ or $2\log(x+a) = 2\log 4a^3$ or $\log(x+a) = \log(16a^6)^{\frac{1}{2}}$	M1
	Removes logs and square roots, **or** halves then removes logs to give $(x+a) = 4a^3$ Or $x^2 + 2ax + a^2 - 16a^6 = 0$ followed by factorisation or formula to give $x = \sqrt{16a^6} - a$	M1
	$(x =)\ 4a^3 - a$ (depends on previous M's and must be this expression or equivalent)	A1cao **(3)**
(ii) Way 1	$\log_3 \dfrac{(9y+b)}{(2y-b)} = 2$ — Applies quotient law of logarithms	M1
	$\dfrac{(9y+b)}{(2y-b)} = 3^2$ — Uses $\log_3 3^2 = 2$	M1
	$(9y+b) = 9(2y-b) \Rightarrow y =$ — Multiplies across and makes *y* the subject	M1
	$y = \frac{10}{9}b$	A1cso **(4)**
Way 2	Or: $\log_3(9y+b) = \log_3 9 + \log_3(2y-b)$ — 2nd M mark	M1
	$\log_3(9y+b) = \log_3 9(2y-b)$ — 1st M mark	M1
	$(9y+b) = 9(2y-b) \Rightarrow y = \frac{10}{9}b$ — Multiplies across and makes *y* the subject	M1 A1cso **(4)** **[7]**

	Notes
(i)	1st M1: Applies power law of logarithms correctly to one side of the equation M1: Correct log work in correct order. If they square and obtain a quadratic the algebra should be correct. The marks is for $x + a = \sqrt{16a^6}$ isw so allow $x + a = \pm\ 4a^3$ for Method mark. Also allow $x + a = 4a^4$ or $x + a = \pm\ 4a^{5.5}$ or even $x + a = 16a^3$ as there is evidence of attempted square root. May see the correct $x + a = 10^{(\log 4 + 3\log a)}$ *so* $x = -a + 10^{(\log 4 + 3\log a)}$ which gains M1A0 unless followed by the answer in the scheme. A1: Do not allow $x = \pm\ 4a^3 - a$ for accuracy mark. You may see the factorised $a(2a + 1)(2a - 1)$ o.e.
(ii)	M1: Applying the subtraction or addition law of logarithms correctly to make **two** log **terms into one** log term in *y* M1: Uses $\log_3 3^2 = 2$ 3rd M1: Obtains **correct** linear equation in *y* usually the one in the scheme and attempts *y* = A1cso: $y = \frac{10}{9}b$ or correct equivalent after **completely correct** work. **Special case:** $\dfrac{\log_3(9y+b)}{\log_3(2y-b)} = 2$ is M0 unless clearly crossed out and replaced by the correct $\log_3 \dfrac{(9y+b)}{(2y-b)} = 2$ Candidates may then write $\dfrac{(9y+b)}{(2y-b)} = 3^2$ and proceed to the *correct* answer – allow M0M1M1A0 as the answer requires a completely correct solution.

Question Number	Scheme		Marks
8. (a)	Way 1	Way 2	
	$1-\sin^2 x = 8\sin^2 x - 6\sin x$	$2=(3\sin x-1)^2$ gives $9\sin^2 x - 6\sin x + 1 = 2$ so $\sin^2 x + 8\sin^2 x - 6\sin x + 1 = 2$	B1
	E.g. $9\sin^2 x - 6\sin x = 1$ or $9\sin^2 x - 6\sin x - 1 = 0$ or $9\sin^2 x - 6\sin x + 1 = 2$	so $8\sin^2 x - 6\sin x = 1-\sin^2 x$	M1
	So $9\sin^2 x - 6\sin x + 1 = 2$ or $(3\sin x-1)^2 - 2 = 0$ so $(3\sin x - 1)^2 = 2$ or $2=(3\sin x-1)^2$ *	$8\sin^2 x - 6\sin x = \cos^2 x$ *	A1cso*
			(3)
(b)	Way 1: $(3\sin x - 1) = (\pm)\sqrt{2}$	Way 2: Expands $(3\sin x-1)^2 = 2$ and uses quadratic formula on 3TQ	M1
	$\sin x = \frac{1\pm\sqrt{2}}{3}$ or awrt 0.8047 **and** awrt -0.1381		A1
	x = 53.58 , 126.42 (or 126.41), 352.06, 187.94		dM1A1 A1
			(5) **[8]**

Notes

(a)

Way 1

B1: Uses $\cos^2 x = 1-\sin^2 x$

M1: Collects $\sin^2 x$ terms to form a three term quadratic or into a suitable completed square format. May be sign slips in the collection of terms.

A1*: cso This needs an intermediate step from 3 term quadratic and no errors in answer and printed answer stated but allow $2=(3\sin x-1)^2$. If sin is used throughout instead of sin*x* it is A0.

Way 2

B1: **Needs correct expansion** and **split**

M1: Collects $1-\sin^2 x$ together

A1*: Conclusion and no errors seen

(b)

M1: Square roots both sides(Way 1), or expands and uses quadratic formula (Way 2) Attempts at factorization after expanding are M0.

A1: **Both** correct answers for sin*x* (need plus and minus). Need not be simplified.

dM1: Uses inverse sin to give one of the given correct answers

1st A1: Need two correct angles (allow awrt) Note that the scheme allows 126.41 in place of 126.42 though 126.42 is preferred

A1: All four solutions correct (Extra solutions in range lose this A mark, but outside range - ignore)

(**Premature approximation**:– in the final three marks lose first A1 then ft other angles for second A mark)

Do **not** require degrees symbol for the marks

Special case: Working in radians

M1A1A0 for the *correct* 0.94, 2.21, 6.14, 3.28

Question Number	Scheme	Marks
9.(a)	$a = 7k-5,\ ar = 5k-7$ and $ar^2 = 2k+10$	B1
	(So $r =$) $\dfrac{5k-7}{7k-5} = \dfrac{2k+10}{5k-7}$ or $(7k-5)(2k+10) = (5k-7)^2$ or equivalent	M1
	See $(5k-7)^2 = 25k^2 - 70k + 49$	M1
	$14k^2 + 60k - 50 = 25k^2 - 70k + 49 \rightarrow\ 11k^2 - 130k + 99 = 0$ *	A1cso * **(4)**
(b)	$(k-11)(11k-9)$ so $k =$	M1
	$k = 9/11$ only* (after rejecting 11)	A1*
	N.B. Special case $k = 9/11$ can be verified in (b) (1 mark only)	
	$11\times\left(\dfrac{9}{11}\right)^2 - 130\times\left(\dfrac{9}{11}\right) + 99 = \dfrac{81}{11} - \dfrac{1170}{11} + \dfrac{1089}{11} = 0$ M1A0	**(2)**
(c)	$a = \dfrac{8}{11}$	B1
	$\dfrac{5\times\frac{9}{11}-7}{7\times\frac{9}{11}-5}$ *or* $\dfrac{2\times\frac{9}{11}+10}{5\times\frac{9}{11}-7}$ so $r = -4$	B1
	(i) Fourth term $= ar^3 = -\dfrac{512}{11}$	M1A1
	(ii) $S_{10} = \dfrac{a(1-r^{10})}{(1-r)} = \dfrac{\frac{8}{11}(1-(-4)^{10})}{(1-(-4))} = -152520$	M1A1
		(6) **[12]**

Notes

(a) Mark parts (a) and (b) together

B1: Correct statement (needs all three terms)**– this may be omitted and implied** by correct statement in k only, as candidates may use geometric mean, or may use ratio of terms being equal and give a correct line 2 without line 1. (This would earn the B1M1 immediately)

M1: Valid Attempt to eliminate a and r and to obtain equation in k only

M1: Correct expansion of $(5k-7)^2 = 25k^2 - 70k + 49$ - may have four terms $(5k-7)^2 = 25k^2 - 35k - 35k + 49$

A1cso: No incorrect work seen. The printed answer is obtained including "=0".

(b) M1: Attempt to solve quadratic by usual methods (factorisation, completion of square or formula – see notes at start of mark scheme) **or** see 9/11 substituted and given as "=0" for M1A0

A1*: 9/11 **only and** 11 should be seen and rejected. Accept 9/11 underlined or k= 9/11 written on following line. Alternatively (k − 11) may be seen in the factorisation and a statement 'k not integer' given with k=9/11 stated.

(c) Mark parts (i) and (ii) together

B1: $a = \dfrac{8}{11}$ or any equivalent (If not stated explicitly or used in formula may be implied by correct answer to (ii))

B1:Substitutes k = 9/11 completely and obtain $r = -4$ (If not stated explicitly, may be implied by correct answer to (i) or (ii))

(i) M1: Use of correct formula with $n = 4$ a and/or r may still be in terms of k or uses $(2k+10)\times r$. May assume $r = k$.

A1: Correct exact answer

(ii) M1: Use of correct formula with $n = 10$ a and/or r may still be in terms of k May assume $r = k$ A1 : -152520 cao

NB Correct formula **with negative sign** in numerator followed by the incorrect $(8/11)(1+4^{10})/(1-(-4))$ usually found equal to 152520.2909 with no negative sign can be allowed M1A0 but if the incorrect numerical expression appears on its own with no formula then M0A0

Listing terms can get: B1 (first term) B1 M1A1 (implied by correct 4th term) M1A1 (implied by −152520)

Question Number	Scheme	Marks
10. (a)	$\frac{dy}{dx} = 12x^2 + 18x - 30$	M1
	Either Substitute $x = 1$ to give $\frac{dy}{dx} = 12 + 18 - 30 = 0$ So turning point (all correct work so far) | Or Solve $\frac{dy}{dx} = 12x^2 + 18x - 30 = 0$ to give $x =$ Deduce $x = 1$ from correct work	A1 A1cso (3)
(b) Way 1	When $x = 1$, $y = 4 + 9 - 30 - 8 = -25$	B1
	Area of triangle ABP = $\frac{1}{2} \times 1 \times 25 = 12.5$ (Where P is at (1, 0))	B1
	Way 1: $\int (4x^3 + 9x^2 - 30x - 8)\,dx = x^4 + \frac{9}{3}x^3 - \frac{30x^2}{2} - 8x\ \{+c\}$ *or* $x^4 + 3x^3 - 15x^2 - 8x\ \{+c\}$	M1A1
	$\left[x^4 + 3x^3 - 15x^2 - 8x\right]_{-\frac{1}{4}}^{1} = (1 + 3 - 15 - 8) - \left(\left(-\frac{1}{4}\right)^4 + 3\left(-\frac{1}{4}\right)^3 - 15\left(-\frac{1}{4}\right)^2 - 8\left(-\frac{1}{4}\right)\right)$ $= (-19) - \frac{261}{256}$ or $-19 - 1.02$	dM1
	So Area = "*their* 12.5" + "*their* $20\frac{5}{256}$" or "12.5" + "20.02" or "12.5" + "*their* $\frac{5125}{256}$"	ddM1
	$= 32.52$ (NOT -32.52)	A1 (7) **[10]**
	Less efficient alternative methods for first two marks in part (b) with Way 1 or 2 For first mark: Finding equation of the line AB as $y = 25x - 50$ as this implies the −25	B1
	For second mark: Integrating to find triangle area $\int_1^2 (25x - 50)\,dx = \left[\frac{25}{2}x^2 - 50x\right]_1^2 = -50 + 37.5 = -12.5$ so area is 12.5 Then mark as before if they use Method in original scheme	B1
(b) Way 2	**Way 2:** Those who use area for **original curve between -1/4 and 2** and subtract area **between line and curve between 1 and 2** have a correct (long) method . The first B1 (if y=−25 is not seen) is for equation of straight line $y = 25x - 50$	B1
	The second B1 may be implied by final answer correct, or 4.5 seen for area of "segment shaped" region between line and curve, or by area between line and axis/triangle found as 12.5	B1
	$\int (4x^3 + 9x^2 - 55x + 42)\,dx = x^4 + \frac{9}{3}x^3 - \frac{55x^2}{2} + 42x\ \{+c\}$ (or integration as in Way 1)	M1A1
	The dM1 is for correct use of the different correct limits for each of the two areas: i.e. $\left[x^4 + 3x^3 - 15x^2 - 8x\right]_{-\frac{1}{4}}^{2} = (16 + 24 - 60 - 16) - \left(\left(-\frac{1}{4}\right)^4 + 3\left(-\frac{1}{4}\right)^3 - 15\left(-\frac{1}{4}\right)^2 - 8\left(-\frac{1}{4}\right)\right)$ **And** $\left[x^4 + \frac{9}{3}x^3 - \frac{55x^2}{2} + 42x\right]_1^2 = 16 + 24 - 110 + 84 - (1 + 3 - 27.5 + 42)$	dM1
	So Area =their $\left[x^4 + 3x^3 - 15x^2 - 8x\right]_{-\frac{1}{4}}^{2}$ **minus their** $\left[x^4 + \frac{9}{3}x^3 - \frac{55x^2}{2} + 42x\right]_1^2$ i.e. "*their* 37.0195" − "*their* 4.5" (with both sets of limits correct for the integral)	ddM1
	Reaching $= 32.52$ (NOT -32.52) See over for special case with wrong limits	A1

	NB: Those who attempt curve – line wrongly with limits −1/4 to 2 may earn M1A1 for correct integration of their cubic. Usually e.g. $\int (4x^3 + 9x^2 - 55x + 42)\,dx = x^4 + \frac{9}{3}x^3 - \frac{55x^2}{2} + 42x \{+c\}$ (They will not earn any of the last 3 marks) They may also get first B1 mark for the correct equation of the straight line (usually seen but may be implied by correct line −curve equation) and second B1 if they also use limits 1 and 2 to obtain 4.5 (or find the triangle area 12.5).	M1A1

	Notes
(a)	M1: Attempt at differentiation - all powers reduced by 1 with $8 \to 0$.
	A1: the derivative must be correct and uses derivative = 0 to find x or substitutes x = 1 to give 0. Ignore any reference to the other root (−5/2) for this mark.
	A1cso: obtains x = 1 from correct work, or deduces turning point (if substitution used – may be implied by a preamble e.g. dy/dx =0 at T.P.)
	N.B. If their factorisation or their second root is incorrect then award A0cso.
	If however their factorisation/roots are correct, it is not necessary for them to comment that −2.5 is outside the range given.
(b)	**Way 1:**
	B1: Obtains y = - 25 when x = 1 (may be seen anywhere – even in (a)) or finds correct equation of line is $y = 25x - 50$
	B1: Obtains area of triangle = 12.5 (may be seen anywhere). Allow −12.5. Accept $\frac{1}{2} \times 1 \times 25$
	M1: Attempt at integration of cubic; two correct terms for their integration. No limits needed
	A1: completely correct integral for the cubic **(may be unsimplified)**
	dM1: We are looking for the start of a correct method here (dependent on previous M). It is for substituting 1 and -1/4 and subtracting. May use 2 and -1/4 and also 2 and 1 **AND subtract** (which is equivalent)
	ddM1 (depends on both method marks) **Correct** method **to obtain shaded area** so **adds** two positive numbers (areas) together – one is area of triangle, the other is area of region obtained from integration of correct function with correct limits (may add two negatives then makes positive)
	Way 2: This is a long method and needs to be a correct method
	B1: Finds y=−25 at x=1 ,or correct equation of line is $y = 25x - 50$
	B1: May be implied where WAY 2 is used and final correct answer obtained so award of final A1 results in the award of this B1. It may also be implied by correct integration of line equation or of curve minus line expression between limits 1 and 2. So if only slip is final subtraction (giving final A0, this mark may still be awarded) So may be implied by 4.5 seen for area of "segment shaped" region between line and curve.
	M1: Attempt at integration of given cubic or after attempt at subtracting their line equation (no limits needed). Two correct terms needed
	A1: Completely correct integral for their cubic **(may be unsimplified) – may have wrong coefficients of *x* and wrong constant term through errors in subtraction**
	dM1: **Use limits for original curve between -1/4 and 2 and use limits of 1 and 2 for area between line and curve–** needs completely correct limits– see scheme- this is dependent on two integrations
	ddM1: (depends on both method marks) Subtracts "*their* 37.0195" − "*their* 4.5" Needs consistency of signs.
	A1: 32.52 or awrt 32.52 e.g. $32\frac{133}{256}$ **NB: This correct answer implies the second B mark**
	(Trapezium rule **gets no marks after first two B marks) The first two B marks may be given wherever seen. The integration of a cubic gives the following M1 and correct integration of their cubic**
	$\int (4x^3 + 9x^2 + Ax + B)\,dx = x^4 + \frac{9}{3}x^3 + \frac{Ax^2}{2} + Bx \{+c\}$ gives the A1

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